AD-A207 027

DTIC
ELECTE
APR 1 4 1989
S D
H

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION: Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY: N/A	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release: distribut[ion] unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE: n/A	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 89-0376
6a. NAME OF PERFORMING ORGANIZATION: West Virginia University	6b. OFFICE SYMBOL (If applicable)
7a. NAME OF MONITORING ORGANIZATION: AFOSR/NM	
6c. ADDRESS (City, State, and ZIP Code): Department of Mathematics, Glennlock Hall, Morgantown, WV 26506	7b. ADDRESS (City, State, and ZIP Code): AFOSR/NM, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR	8b. OFFICE SYMBOL (If applicable): NM
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-87-0347	
8c. ADDRESS (City, State, and ZIP Code): Building 410, Bolling AFB, DC 20332	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO.
61102F	2304	A 9	

11. TITLE (Include Security Classification)

Problems in Dynamic Phase Transitions

12. PERSONAL AUTHOR(S)

Dr. Hattori

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM 1 Sep 87 TO 30 Sep 88	18 Nov 88	5

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Under this grant the following paper has been acepted for publication in the new journal edited by George Sell: Mischaikow, K., & H. Hattori, On the existence of intermediate magnetohydrodynamic shock waves. In this paper the authors have used the technique called the connection matrix to establish one of the intermediate shocks which has not been shown to exist. This technique is also applied to the above project.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: DR. ARJE NACHMAN	22b. TELEPHONE (Include Area Code): 202-767-4939 / 22c. OFFICE SYMBOL: NM

DD Form 1473, JUN 86 — Previous editions are obsolete. — SECURITY CLASSIFICATION OF THIS PAGE: UNCLASSIFIED

89 4 13 150

Report AFOSR-87-0347

Problems in Dynamic Phase Transition

Harumi Hattori
Department of Mathematics
West Virginia University
Morgantown, WV 26506

November 18, 1988

Final Report

Reference No 87-222

Summary of Project

There are a few approaches to the phase transition problem. One is to consider the momentum equation and another one is to consider the energy equation. The simplest in the first approach is the wave equation with nonmonotone constitutive relation, namely,

$$u_{tt} = \sigma(u_x)_x, \tag{1}$$

where u is displacement and σ is given in Fig. 1. There is a difficulty for this approach. That is, when $\sigma' < 0$, the system is elliptic and may not be well posed. The "Hadamard instability" may take place. One way to overcome this difficulty is to add regularizing terms which regularize (1). Dafermos [1] and Pego [2] considered the effect of viscosity, namely,

$$u_{tt} = \sigma(u_x)_x + \nu u_{xxt}. \tag{2}$$

Slemrod [3] considered the regularization of (1) by the viscosity and the capillarity, namely,

$$u_{tt} = \sigma(u_x)_x + \nu u_{xxt} - \eta u_{xxxx}. \tag{3}$$

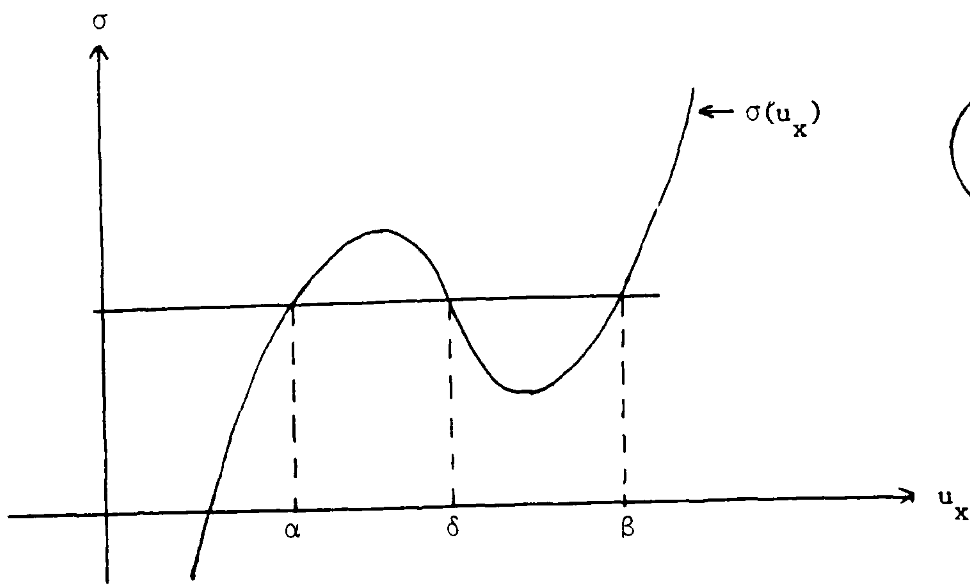

Figure 1.

In this proposal I have considered the questions associated with the equation (3), which has the following motivation. Carr, Gurtin, and Slemrod [4] considered the variational problems associated with (3). Specifically, they considered the following problem.

($\mathfrak{P}$) Minimize

$$\int_{-1}^{1}[W(v) + \eta v'(x)^2/2 - Pv]dx$$

over all $v \in H^1(-1,1)$.

In ($\mathfrak{P}$) $W(v)$ is a primitive of $\sigma(v)$. They have shown that depending on the value of η and P there exist multiple solutions. When $\sigma_* < P < \sigma^*$, there are only constant solutions α, δ, and β if $\eta > C$ (a positive number), and among them α and β are the local minimizers of ($\mathfrak{P}$) and δ is not the local minimizer. When $\sigma_* < P < \sigma^*$ and $\eta < C$, there are nonconstant solutions besides the above constant solutions, but none of nonconstant solutions are the minimizers of ($\mathfrak{P}$).

For ($\mathfrak{P}$) a dynamic problem is

$$u_{tt} = \sigma(u_x)_x + \nu u_{xxt} - \eta u_{xxxx}, \tag{3}$$

$$u(0,t) = 0, \; \sigma(u_x) + \nu u_{xt} - \eta u_{xxx}|_{x=1} = P, \tag{4a}$$

$$u_{xx}(0,t) = 0, \; u_{xx}(1,t) = 0. \tag{4b}$$

Here (4a) corresponds to the soft loading device in which the stress at $x = 1$ is kept at P, and (4b) are the natural boundary conditions to ($\mathfrak{P}$). Using the transform

$$p = \int_{-1}^{1} u_t dx, \; q = u_x$$

used by Pego [2] we obtain the initial boundary value problem

$$\begin{aligned} p_t &= \nu p_{xx} - \eta q_{xx} + \sigma(q) - P, \\ q_t &= p_{xx}, \end{aligned} \tag{5}$$

$$p_x(0,t) = 0, \; p(1,t) = 0, \tag{6a}$$

$$q_x(0,t) = 0, \; q_x(1,t) = 0. \tag{6b}$$

The advantage of this transform is that the system (5) is semilinear and the boundary conditions are linear. Therefore, we can apply the semigroup theory. The questions and results concerning the above initial boundary value problem is summarized in the following way.

Results

(i) As stated in Summary of Project, the constant solutions α and β are stable in the variational problem, but it is not clear if they are dynamically stable. I was able to show the dynamical stability of the constant solutions α and β, the instability of the constant solution δ and the nonconstant solutions.

(ii) Another interesting question is that when there are multiple steady state solutions, there may be solutions connecting them. These special solutions are called connecting orbits. I was able to show the existence of such solutions. This will provide us informations on the dynamical behavior of solutions.

These results will be submitted for publication in the near future.

Significance of The Results

The existence of connecting orbits from δ to α and β will help understand the dynamical behavior of solutions. Namely, it shows that if we set the initial data close to $(p,q) = (0,\delta)$, solution will eventually approach $(p,q) = (0,\alpha)$ or $(0,\beta)$. It also tells us the effect of the viscosity and the capillarity terms, because we do not know how solutions to (1) behave in the region where σ' is negative.

The connecting orbit problem for semi-flow has been discussed for a single parabolic equation, but not for systems. It is important to consider the connecting orbit problem for the semi-flow in systems.

This is a first step toward such a direction.

References

[1] Dafermos, C.M., The mixed initial-boundary value problem for the equations of nonlinear one dimensional viscoelasticity, J. Diff. Eqns. 6 (1969), 71-86.

[2] Pego, R., Phase transitions: Stability and admissiblity in one dimensional nonlinear viscoelasticity, IMA Report #180 (1985).

[3] Slemrod, M., Admissibility criteria for propagating phase boundaries in a van der Waals fluid, Arch. Rat. Mech. Anal. 5 (1982), 345-359.

[4] Carr, J., M.E. Morton, & M. Slemrod, One dimensional structured phase transformations under prescribed loads, J. Elasticity 15 (1985), 133-142.

Publications

Under this grant the following paper has been accepted for publication in the new joural edited by George Sell:

Mischaikow, K., & H. Hattori, On the existence of intermediate magnetohydrodynamic shock waves.

In this paper the authors have used the technique called the connection matrix to establish one of the intermidiate shocks which has not been shown to exist. This technique is also applied to the above project.

MASS

Am

I contacted Dr. Mischaikow about the title of the journal but he did not remember it.

Professional Presentations

In this period the following presentation was made:

The Reimann problem for a van der Waals fluid and the entropy rate admissiblity criterion, June 1988 University of Wisconsin.